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CLASSIFYING THE QUANTUM PHASES OF MATTER

CALIFORNIA INSTITUTE OF TECHNOLOGY

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13. SUPPLEMENTARY NOTES

14. ABSTRACT

This is the final report for "Classifying the Quantum Phases of Matter," FA8750-12-2-0308. Among other achievements, we made progress toward the following goals: (1) finding a classification of locally definable quantum states (those without symmetries or long-range entanglement), (2) elucidating the properties of three-dimensional quantum codes (in particular those which admit no string-like logical operators. (3) characterizing symmetry-protected fermionic phases, (4) proving the stability of topological phases of matter with respect to generic perturbations.

15. SUBJECT TERMS

quantum topological phases, quantum codes, many-body quantum states

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1.0 SUMMARY

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2.0 INTRODUCTION

Recent advances in quantum information theory have ignited a quest for a “grand unified theory” of quantum many-body physics. This quest, if successful, may yield far-reaching answers to the following questions: What many-body quantum states can be ground states of physically realizable Hamiltonians? What universal features of these states are robust when the Hamiltonian is slightly deformed? How is this classification modified when the Hamiltonian is required to respect certain symmetries? How are bosonic systems different from fermionic systems, and how are interacting systems different from free systems? What properties of many-body quantum entanglement distinguish one quantum phase from another?

Aside from their fundamental importance, these questions relate to the AFRL/DARPA mission because of their implications for future quantum technologies. For example, new quantum phases of matter may lead to better ways to protect quantum information from damage caused by decoherence, and to process it reliably using imperfect hardware. Further theoretical progress may also elucidate the complexity of quantum simulation tasks, pointing toward a better characterization of which simulation problems are likely to be hard for classical computers, yet feasible for quantum computers.

One central goal of this project has been to understand topological properties of quantum phases, describe them by a suitable mathematical structure, and classify them. We also investigated the classification of quantum phases that are protected by global symmetries, studied properties of quantum codes with local check operators in three dimensions, and investigated the structure of quantum entanglement in gapped quantum phases.

The participants in this project were:

Faculty: Alexei Kitaev, John Preskill

Postdocs: Andrew Essin, Zhengcheng Gu, Spiros Michalakis, Fernando Pastawski, Beni Yoshida

Students: Michael Beverland, Jeongwan Haah, Isaac Kim, Alex Kubica, Sujeet Shukla

3.0 METHODS, ASSUMPTIONS AND PROCEDURES

3.1 Locally definable states

Superconductors, topological superconductors, and integer quantum Hall phases can all be realized in systems of noninteracting (“free”) fermions. In 2009, Kitaev used K-cohomology to completely classify all gapped phases of free fermions in any dimension, and in 2010 Fidkowski

and Kitaev showed that interacting spins or fermions in one dimension can be classified using the cohomology of the system's symmetry group. The remaining challenge is to generalize this classification to systems with interactions in two or more dimensions.

The general classification problem is quite difficult, but a complete understanding of quantum states without long-range entanglement may be within reach. Such states may be called ``locally definable" because they are uniquely specified by the reduced density matrices on small subsystems. A major goal of our work has been to classify such states. One may anticipate a close connection with the theory of "anomalies" in relativistic quantum field theory, with possible differences due to the lack of Lorentz symmetry.

3.2 Fractal topological order

We say a quantum system is topologically ordered if the information encoded in the system's global state is inaccessible to local observers. What are believed to be topologically ordered states of two-dimensional matter have been studied in the laboratory (fractional quantum Hall states, for example), but less is known about the possible types of topological order in three dimensions.

In 2011, Preskill's student Jeongwan Haah discovered a novel mathematical model exhibiting three-dimensional topological order. Haah's model can serve as a robust quantum memory, such that logical errors in its stored quantum information arise only if noise excites the system enough to surmount an energetic barrier which grows logarithmically with the system size. Although Haah's model is translation invariant, it nevertheless behaves like a spin glass; though the system supports pointlike quasiparticles, the propagation of the particles is impeded. Movement of a single particle requires a complex self-similar (fractal) process, in which many additional particles are successively created and annihilated. Another goal of our work has been to place the Haah model in context, better understand its properties, and explore related models exhibiting exotic topological order in three dimensions.

4.0 RESULTS AND DISCUSSION

4.1 Classification of three dimensional topological superconductors

Kitaev has classified three-dimensional time-reversal invariant fermionic systems with short-range entanglement, also known as topological superconductors [14]. The system is characterized by an by an integer invariant ν, the number of gapless fermion species residing on the two-dimensional surface of the three-dimensional bulk sample. Kitaev showed that (for time-reversal invariant systems such that $T^2 = -1$) when ν is a multiple of 16 a gap can be opened by a suitable perturbation that preserves T symmetry and produces no long-range entanglement, and that in that case the bulk phase can be adiatically transformed to a trivial phase.

More generally, Kitaev formulated a definition of quantum phases with short-range entanglement: a state is short-range entangled if we can combine the state with a suitably chosen conjugate state, and then map the combined state to a trivial product state using a constant depth quantum circuit. He proposed a topological classification of all such phases in any dimension,

which can be reduced to the computation of a topological invariant of the system. In the three-dimensional case this computation can be carried our explicitly [Fig. 1].

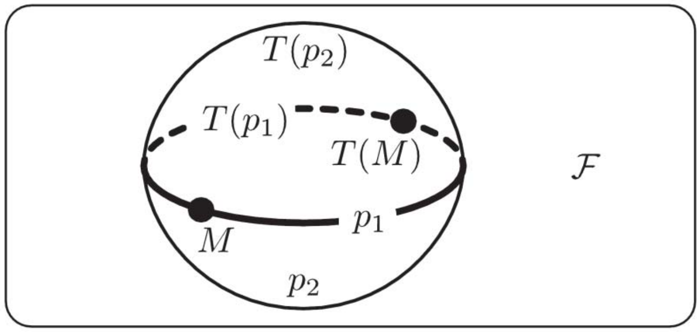

Fig 1. Analysis of topological obstructions protecting gapless fermions at surface of topological superconductor.

4.2 Renormalization group flow of three-dimensional code states

Haah studied the renormalization group flow of three-dimensional code states, in order to better understand the long-range entanglement of these states [17], and in particular used the renormalization group to investigate the quantum memory model discovered earlier by Haah. He defined a block spin procedure for these codes, which maps the ground state of a local Hamiltonian H_A on a lattice with spacing a to two uncoupled ground states of Hamiltonians H_A and H_B on a lattice with spacing $2a$. Furthermore, applying the block spin transformation to the ground state of H_B yields two copies of that ground state.

These results clarify the origin of the extensive ground-state degeneracy of Haah's model, as well as providing new tools for performing entanglement renormalization in topologically ordered systems, and finding tensor network descriptions of highly entangled states. Studying the dependence of degeneracy on system size reveals that the ground states of H_A and H_B represent distinct phases of matter.

4.3 Summary of other results (synopsis of publications)

Here, briefly summarized, are some of the other outcomes of this project.

Isaac Kim derived of a new entropic inequality, which generalizes strong subadditivity to an operator setting [1]. Kim explored some of the consequences of his new inequality, deriving in particular new universality properties for the entanglement spectrum for a subsystem of the ground state of a local Hamiltonian [2,3].

Kitaev et al. showed that the ground state energy of a gapped local one-dimensional quantum system can be computed in subexponential time [4].

Michalakis et al. studied Markovian dynamics governed by local Lindblad operators, showing that local observables and correlation functions are stable with respect to generic local perturbations if the mixing time scales logarithmically with the system size [6].

Yoshida constructed three-dimensional models which are topologically ordered but cannot be described by conventional topological quantum field theory because of the fractal structure of the quantum ground state [5].

Gu used braiding statistics to classify the two-dimensional symmetry protected topological phases with Ising symmetry [7].

Kim related entanglement entropy to topological storage of quantum information [8].

Michalakis et al. showed that a particle-like excitation spectrum is a characteristic property of gapped translation-invariant local systems, and that topologically ordered projective entangled pair states are robust with respect to local perturbations [9,12], (see Fig. 2).

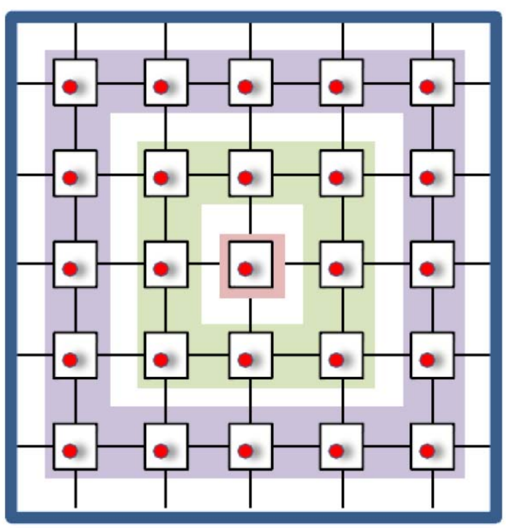

Fig. 2. Verifying local topological quantum order in tensor network states.

Haah studied the algebraic properties of three-dimensional quantum code states [10].

Gu et al. described a simple fermionic version of the toric code model which illustrates many features of fermionic exactly solvable models and fermionic topologically ordered states. Their

work shows how topological phases of matter are richer for fermionic systems than for their bosonic counterparts [13].

Pastawski studied the time needed to prepare D-dimensional topologically ordered states using Markovian open system dynamics, showing that the time scales like the diameter of the system [15,16].

Michalakis et al studied information flow in quantum systems with long-range interactions, deriving a limiting flow velocity, which has since been verified in ion trap experiments [18,19].

Kubica and Yoshida developed a novel real-space renormalization group scheme which accurately estimates the correlation length exponent scaling exponent near criticality of higher-dimensional quantum Ising and Potts models in a transverse field [20].

Yoshida showed that the storage time of a classical or quantum memory can be enhanced even though the memory is disordered by thermal fluctuations [21].

Yoshida and Kubica studied the quantum Ising model on a fractal lattice, showing that the universality class of the quantum phase transition is not uniquely determined by the symmetry and spatial dimension of the system [22].

Pastawski and Yoshida proved a no-go theorem for self-correcting quantum memory, showing in particular that a three-dimensional stabilizer Hamiltonian with a locality preserving implementation of a non-Clifford gate cannot have a macroscopic energy barrier [23].

Beverland et al. studied code automorphisms induced by locality-preserving unitary transformations, finding these logical operations are very limited in codes supporting non-abelian anyons [24].

Essin et al. developed a new numerical method for detecting symmetry enriched topological phases [25].

5.0 CONCLUSIONS

This project has illuminated fundamental properties of quantum phases of matter that can be realized as ground states of local Hamiltonians or fixed points of stochastic processes. We formulated a classification of three-dimensional phases with short-range entanglement, characterized quantum states that can be prepared by dissipative processes, described the logical quantum gates that can be performed by locality preserving transformations, and clarified the properties of topologically ordered systems at nonzero temperature.

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7.0 LIST OF SYMBOLS, ABBREVIATIONS, AND ACRONYMS

None were used in this report.